Worksheet 5

1. Consider the matrices

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}.$$

   For what value(s) of k, if any, do matrices A and B commute?

2. Suppose P is an invertible matrix, and $A = PBP^{-1}$, where A and B are also matrices. Solve for B in terms of A.

3. Find the inverse of the following matrix, if it exists:

$$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

4. Answer the following short questions, justifying your answers. Note that all referenced matrices are $n \times n$ square.

   (a) Matrix A has one column that is 7 times another column. Is A invertible?

   (b) A is the standard matrix of a linear transform $T : \mathbb{R}^n \to \mathbb{R}^n$ that is one-to-one. Is A invertible?

   (c) A is not invertible. How many solutions are there to the equation $A\vec{x} = \vec{0}$?

   (d) There are some vectors in $\mathbb{R}^n$ that are not in the span of the columns of A. Is A invertible?