Math 1552, Integral Calculus
Sections 10.2-10.8

1. Terminology review:
complete the following statements.
(a) A geometric series has the general form ______________. The series converges when ______________ and diverges when ______________.

(b) A p-series has the general form ______________. The series converges when ______________ and diverges when ______________. To show these results, we can use the ______________ test.

(c) The harmonic series ______________ and telescoping series ______________.

(d) If you want to show a series converges, compare it to a ______________ series that also converges. If you want to show a series diverges, compare it to a ______________ series that also diverges.

(e) If the direct comparison test does not have the correct inequality, you can instead use the ______________ ______________ test. In this test, if the limit is a ______________ number (not equal to ______________), then both series converge or both series diverge.

(f) In the ratio and root tests, the series will ______________ if the limit is less than 1 and ______________ if the limit is greater than 1. If the limit equals 1, then the test is ______________.

(g) If $\lim_{n\to\infty} a_n = 0$, then what do we know about the series $\sum_n a_n$? ______________

(h) A power series has the general form: ______________. To find the radius of convergence R, use either the ______________ or ______________ test. The series converges ______________ when $|x-c| < R$. To find the interval of convergence, don't forget to check the ______________. If we differentiate or integrate a power series, the radius of convergence of the new series is ______________.

(i) A Taylor Polynomial has the general form: ______________.

2. Sum the series

$$\sum_{k=2}^{\infty} \frac{4^{2k}-1}{17^{k-1}}.$$

3. Find the radius and interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{5^k}{\sqrt{k}}(3-2x)^k.$$

4. Find the third degree Taylor polynomial of the function $f(x) = \tan^{-1}(x)$ in powers of $x-1$.

5. Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)}.$$

6. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

a) $\sum_{k=1}^{\infty} \frac{e^k}{(1+4e^k)^{3.2}}$

b) $\sum_{k=2}^{\infty} \left(\frac{k-5}{k}\right)^{k^2}$

c) $\sum_{k=1}^{\infty} \frac{k^2 \cdot 2^{k+1}}{k!}$

d) $\sum_{k=1}^{\infty} \frac{1}{1+2+3+\ldots+k}$

7. Use the MacLaurin series for $f(x) = \frac{1}{1-x}$ to find a power series representation of the function

$$g(x) = \frac{x}{(1-x)^3}.$$

HINT: You will need to differentiate.

8. Determine whether each of the alternating series below converge absolutely, converge conditionally, or diverge. Use the convergence tests from class to justify your answer.

a)

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\ln k}{k^4}$$

b)

$$\sum_{k=2}^{\infty}(-1)^{k}\frac{4k^2}{k^3+1}$$

9. Find a power series (MacLaurin series) for each function below.

a) $f(x) = \frac{x}{4+x^4}$

b) $g(x) = x\sin(x^2)$

ANSWERS

2. $255\frac{15}{16}$

3. $R = \frac{1}{10}$, $I.C. = \left(\frac{7}{5}, \frac{8}{5}\right]$

4. $P_3(x) = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

5. $\frac{1}{3}$

6. (a) Converges by the integral test OR a direct comparison with the appropriate geometric series
(b) Converges by the root test
(c) Converges by the ratio test
(d) Converges by direct comparison (HINT: you will need to use the formula $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ to simplify the expression first) OR you can reduce to show it is telescoping

7. $\frac{x}{(1-x)^3} = \frac{1}{2}\sum_{k=2}^{\infty} k(k-1)x^{k-1}$, valid when $|x| < 1$

8. (a) Converges absolutely (can use direct comparison or the integral test on the series of absolute values)
(b) Converges conditionally (use the limit comparison test on the series of absolute values, followed by AST)