Orthogonal projection

Find A so that T_A is orthogonal projection onto

$$W = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \right\}$$

Find B so that T_B is orthogonal projection onto

$$L = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$$

Answer the following questions (without calculation!).

1. What are A^2 and B^2?
2. What are A^{-1} and B^{-1}?
3. What are AB and BA?
4. Is A or B diagonalizable?
5. What are the eigenvalues of A and B (with algebraic multiplicity)?
6. Is A similar to B?

Best approximation

W = subspace of $\mathbb{R}^n$

Fact. The projection y_W is the point in W closest to y. In other words:

$$\|y - y_w\| < \|y - w\|$$

for any w in W other than y_w.

Why?

Best approximation

Problem. Find the distance from e_1 to $W = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$.

Find a best "solution" to $Ax = e_1$ where

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$$